Let $u^2=\frac{128}{81}$. Then $3m=5$ or $m=\frac{5}{3}$, $n=\frac{4}{3}$, $x=3$.

$\therefore$ $mx=5$, $nx=4$, $x=3$. Let $u^2=4$. Then $m=1.332$, $n=.8799$, $x=.936-$. $\therefore$ $mx=1.246$, $nx=.8235$, $x=.936$, when $a=2$, $b=1$.

Let $a=40$, $b=50$, $u^2=\frac{16}{25}$, Then $m=1.2532$, $n=.7553$, $x=47.4072+$.

$\therefore$ $mx=59.4107+$, $nx=35.8067+$, $x=47.4072+$.

Let $a=b=c$, then $u^2=1$, $m=\sqrt{2}$, $n=1$, $x=\frac{c}{2}\sqrt{(2+\sqrt{2})}$.

$$\therefore \quad mx=\frac{c}{\sqrt{2}}\sqrt{(2+\sqrt{2})},\ nx=x=\frac{c}{2}\sqrt{(2+\sqrt{2})}.$$

III. **Solution by B F. BURLESON, Oneida Castle, New York**

Let ABC be the triangle, right angled at C. Put $AD=a=40$, and $BE=b=50$, the lines bisecting the acute angles A and B. Put $x=AB$, $y=AC$, and $z=BC$. Put $\phi+\theta=$the$\angle CAD$ and $\phi-\theta=$the $\angle CBE$. We have, by Trigonometry,

$$x=b\cos(\phi-\theta),\ldots.(1),$$
$$y=a\cos(\phi+\theta),\ldots.(2),$$
$$y=z\tan(2\phi-2\theta)\ldots.(3),$$

Eliminating from (1), (2), and (3), we obtain by development

$(b+b\tan\phi\tan\theta)\left(\dfrac{1-\tan^2\theta-2\tan\theta}{1-\tan^2\theta+2\tan\theta}\right)=0\ldots.(4)$. This is true because $\phi=22\frac{1}{2}^{\circ}$.

Clearing (4) of fractions, resolving factors, and substituting for $\tan\phi=22\frac{1}{2}^{\circ}$ its equal $\sqrt{2}-1$, observing that $\cot\phi=\sqrt{2}+1$, we get $(b+a)\tan^2\theta+\{(b-a)(\sqrt{2}+1)+[2(b-a)]\}\tan^2\theta+\{2(b+a)(\sqrt{2}+1)-(b+a)\}\tan\theta=(b-a)(\sqrt{2}-1)\ldots.(5)$. Dividing (5) by $b+a$ and substituting the numerical values of a and b, we get

$\tan^3\theta-490468\tan^2\theta+3.828427125\tan\theta=.268245951375$. Hence, by Horner's Method of Detached Coefficients, $\tan\theta=.0693633$, and the auxiliary angle $\theta=3^{\circ}58'4\frac{7}{8}''$. By substituting in (1) and (2), we determine that $y=35.807338$ and $z=47.407325$. $\therefore$ $x=\sqrt{(y^2+z^2)}=59.410604$.

This problem was also solved by *A. H. Bell*, *J. F. W. Scheffer*, and *H. C. Wilkes*.

42. **Proposed by ALEXANDER MACFARLANE, A. M., D.Sc, LL. D., Cornell University, Ithaca, New York.**

There are p electors and q candidates for r seats. Each elector has r votes, and he may distribute them as he pleases among the candidates. Find in how many different ways the voting may result, that is, the number of possible states of the poll.

Solution by G. B. M. ZERR, Staunton, Virginia, and F. P. MATZ, New Windsor, Maryland.

The number of different ways of voting for r seats out of q candidates, when each elector casts r votes for r different persons, is

$$n=\frac{q(q-1)(q-2)(q-3)\ldots.(q-r+1)}{1.2.3.4\ldots.r}.$$

If $p>n$, then, since there can be but n different ways of voting, n will be the number of different ways the voting may result.

If $p<n$, then since p persons can prepare only p states of the poll, p will be the number of different ways the voting may result.

Also solved by *H. C. Whitaker*.

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

40. Proposed by J. C. CORBIN, Pine Bluff, Arkansas.

If R, r, r_1, r_2, and r_3 be, respectively, the radii of the circumscribed, inscribed, and escribed circles of a $\triangle$, prove $r_1+r_2+r_3-r=4R$.

Solution by M. A. GRUBER, War Department, Washington, D. C.

From any $\triangle$ whose sides are a, b, and c, we obtain $R=\dfrac{abc}{4\triangle}$,

$r=\dfrac{\triangle}{s}$, $r_1=\dfrac{\triangle}{s-a}$, $r_2=\dfrac{\triangle}{s-b}$, and $r_3=\dfrac{\triangle}{s-c}$.

$$\therefore\quad r_1+r_2+r_3-r=\frac{\triangle}{s-a}+\frac{\triangle}{s-b}+\frac{\triangle}{s-c}-\frac{\triangle}{s}=\frac{2s^3-as^2-bs^2-cs^2+abc}{\triangle}$$

$$=\frac{s^2[2s-(a+b+c)]+abc}{\triangle}=\frac{abc}{\triangle}.\quad \text{But } \frac{abc}{\triangle}=4R.\quad \therefore\quad r_1+r_2+r_3-r=4R.$$

We might appropriately add a few other combinations of these radii.

$$(1)\ \frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r};\ (2)\ rr_1r_2r_3=\triangle^2;\ (3)\ Rrr_1r_2r_3=\frac{abc\triangle}{4}.$$

Solutions of this problem were received from *G. I. Hopkins*, *E. W. Morrell*, *P. S. Berg*, *G. B. M. Zerr*, *F. P. Matz*, *Cooper D. Schmitt*, *P. H. Philbrick*, *J. F. W. Scheffer*, *John B. Faught*, and the *Proposer*. H. C. Whitaker did not solve the problem but referred to Chauvenet's Geometry and Hallowell's Geometrical Analysis, p. 225.

41. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find the length (x) of a rectangular parallellopiped $b=5$ft., and $h=3$ft., which can be *diagonally inscribed* in a similar parallelopiped $L=83$ft., $B=64$ft, and $H=50$ft.